Academic Life before, at, and after IISc Some Reminiscences



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Before going up to IISc in 1963, I spent perhaps the most enlightening time in Sarada Vilas High School, Mysore, where I found myself in the challenging, but refreshing, company of students who, in stark contrast with me, came from special private schools meant for the society's elite in those halcyon days of royal Mysore. It is impossible to convey the pain I now experience over the palpable loss of Mysore's grandeur, a grandeur originally attributed to the royal family. Before I landed in the only engineering college of Mysore, I passed through the portals of Sarada Vilas College (for the Pre-University program). My greatest teacher in high school and college was my father who had been educated in England (Manchester).

The under-graduate engineering program (with specialization in Electrical Engineering) was nightmarish, bordering on veritable wilderness. But this period became, by and large, bearable, thanks *mainly* to the Government of India's renewed Merit Scholarship, which had been first granted to me in high school earlier and continued in Sarada Vilas College, too. If only to give a glimpse into the uneasy (to me) atmosphere prevailing in the engineering college, I cannot resist mentioning just one event, an apparently traditional one — a farewell party — that took place at the end of a year. The then-head of a department (whose only expertise consisted of reproducing or, rather, regurgitating verbatim, material from a standard book) remarked maliciously and out of context that I was "earning while learning", as though his finances were being drained, and seemed to grin with satisfaction over his choice of phrase. I hasten to add that such an insensitive and deeply hurtful utterance by the head of a department in a college was not endemic to an undergraduate institution. Fast forward, something similar, and of a more serious nature, was typical of events in an established, higher place of learning, too.¹ One may dismiss such events as inconsequential,² but they aren't.

1 Arrival at the Indian Institute of Science

I was exuberantly relieved in May/June 1963 when I learned that I had qualified for the award of the Bowen Memorial Prize of the University of Mysore. Soon thereafter, I "arrived" at the Institute of Science, Bangalore to find myself in an atmosphere of intoxicating delight, and I revelled in it. The companionship was stimulating, and a few teachers (especially in Mathematics) were masters of their fields of activity with an uncanny ability to provoke thought while eliciting fun out of solving even standard problems on the blackboard with no notes in hand (unlike in the engineering college that, as I now strongly feel, I *managed* somehow to come out of).

My research baptism at the Institute was presided over by Professor B. L. Deekshatulu under whom I wrote up a couple of papers on the analysis of linear, finite dimensional systems with (i)

¹ See item (1) in the section "Notes" at the end of the article.

²In my view, they do affect the idealistic view a sensitive mind conceives of a justiably revered research haven.

a time delay, and (ii) time-varying parameters.³ As far as the first topic is concerned, the only accessible material at that time (in the library of the Institute) amounted to (a) the approximate results of N. Minorsky (gleaned from his voluminous, but classic, book on nonlinear oscillations); and (b) R. Bellman and R. Cooke's treatise (on differential-difference equations). The latter, with its theorems on existence and uniqueness conditions, was indeed terrifying, trained as I was in applied (or, rather, engineering) mathematics (where very little is taught in terms of existence and uniqueness of solutions to differential equations and their relevance to research investigations). In fact, in the mathematics classes (even in the Institute) at that time, no reference had been made to, for instance, E. Coddington and N. Levinson's standard book. And, it also should be added, there had been no formal course on real analysis as such in the Institute. (In fact, the instructors were literally from the Department of Applied Mathematics, as there was no separate Department of Pure Mathematics.) Be that as it may, I came out with a new, non-standard expansion technique to obtain a recursive solution to a linear differential-difference equation with a small parameter, and demonstrated its practical usefulness by hand-computation using an old *cranking mechanical calculator* installed in the corner of a big room occupied by Professor BLD. (It is to be noted that the nearest access point to a computer was TIFR Bombay, and programming that computer was far beyond my field of view.) The paper with the proposed expansion appeared in an international journal.⁴

1.1 Feedback System Stability

Disenchanted as I was with the preliminary work on linear time-varying system analysis which was, much to my surprise, but rather alloyed excitement, duly published in some international journals, I turned to a more topical and challenging problem which concerned the stability analysis of feedback systems with a linear time-invariant forward block and linear/nonlinear time-invariant/time-varying gain in feedback.⁵ Partial *post factum* motivation arose from the intriguing behaviour of a solution of a linear second-order differential equation with a periodic coefficient having two parameters, *a* and fl, viz. the Mathieu equation, $d^2 x$

$$-TTT + (a + fl \cos 2t)x(t) = 0, t E [0, to); a > 0, fl > 0.$$
(1)
$$dt^{2}$$

This equation has been known for a long time, and a remarkable monograph [1],⁶ which is entirely devoted to it, contains fl-vs-a graphs of stability-instability boundaries, obtained from series expansions (in terms of the so-called Mathieu functions). In effect, McLachlan employs a *quantitative* approach to stability/instability, as against the current trend of *qualitative* methods which have been developed mainly for the stability analysis of higher-order and more *general*, including nonlinear, systems.⁷ It is known that these McLachlan-graphs can be employed as a benchmark in the analysis ³⁴⁵⁶⁷

³In retrospect, the results for the latter are inconsequential, while those for the former are inadequate, incomplete and unsatisfactory.

⁴Many years later, I saw its review, in a review journal, by, if I remember correctly, R. E. Kalman who very rightly criticised its numerical results.

⁵Many single-input-single-output (SISO) systems can be cast into this form, at least as an approximation; others, into a multi-input-multi-output (MIMO) form with the same feedback structure. The latter's stability conditions became the subject of my interest later.

⁶See references at the end of the article.

⁷See below for an outline of the more general problems.

of, for instance, the stability of oscillations in nonlinear systems by suitable approximation techniques. Moreover, there are physical systems (like the parametric amplifiers) which are inherently time- varying and periodic, too.

An extraordinary property of Eqn. (1) is that whatever general techniques (of which the direct method of Lyapunov, in its original form or appropriately generalized, is one) have been developed to arrive at stability conditions for second- and higher-order linear (and nonlinear) time-varying systems *cannot*, when applied to Eqn. (1), reproduce those (for want of a better expression) McLachlan stability-instability boundaries.⁸ It is helpful to note here that Eqn. (1) can be recast in such a form that it describes the behaviour of a feedback system with a (linear) time-invariant-block in the forward path and the time-varying gain $(a +)3 \cos 2t$) in feedback. A minor generalization of this leads first to an arbitrary time-varying gain, k(t) > e > 0, $t \in [0, ro)$, in feedback.

Stage-by-stage further generalizations of Eqn. (1) lead us to the following systems: (a) a feedback gain whose output is k(t)p(x), where <^(-) is the first-and-third-quadrant nonlinearity (satisfying certain assumptions); and with the same feedback gain k(t) > e > 0, t E [0, ro), (b) a higher-order (i.e., > 2) finitedimensional (time-invariant) linear forward block having the rational transfer function G(s); and (c) an infinite-dimensional (time-invariant) linear forward block (representing, for instance, distributed-parameter systems) having, in general, not necessarily rational transfer function G(s). It is interesting to note that, even for the above case (a), stability-instability boundaries (extending those found in [1] are not known. Paradoxically, while many stability results⁹ are known for Cases (b) and (c)¹⁰ using Lyapunov or other frequency-domain-based methods (of which Popov's [2] is the most original) or even Zames's *positive-operator* theory [3], none of them can, in the (limiting or) special case¹¹ of Eqn. (1), lead anywhere close to the stability boundaries found in [1].

For general systems mentioned in cases (b) and (c) above, when we replace the nonlinear time- varying feedback gain by merely a linear constant gain, the frequency-domain stability conditions, which are necessary and sufficient, are attributed to Nyquist, and, as is well known, these conditions are, for finite-dimensional systems, equivalent to those that can be obtained from the Routh-Hurwitz criterion. What is, however, not immediately apparent is that the Nyquist criterion, for *both* finite- and infinite-dimensional systems, can be cast in terms of a *multiplier-function*, Z(ju) in the frequency domain: The system corresponding to cases (b) and (c) above, but with a constant feedback gain $K \in [0,K)$, is asymptotically stable, if there exists a frequency function, Z(ju) such that $-n/2 < \arg\{Z(ju), (G(ju) + ==)\} < n/2$, where "arg" denotes "the phase angle of". Alternatively, !RZ(ju) > 0, and !RZ(ju)(G(ju) + ==) > 0, u E (—ro, ro), where " !R" denotes "the real part of". One of the goals is to derive stability criteria for (linear and) nonlinear (time-invariant and) time-varying gain feedback systems that reduce to the benchmark Nyquist criterion when the feedback gain is linear and constant.

An important result attributed to many researchers is the circle criterion: A nonlinear time- varying feedback system is stable¹² if the multiplier function Z(ju) is simply unity, i.e., -n/2 <

⁸See item (2) in *Notes* at the end of the article.

⁹One should mention here a few instability results obtained mainly in the Lyapunov framework.

¹⁰Note that Case (c) includes Case (b) as a special case.

¹¹ More correctly, its slightly modified version having some (arbitrarily small) damping to facilitate the formal applicability of the existing qualitative methods.

¹²This is a misuse of terminology. In a Lyapunov framework, this implies, under certain conditions the system is

 $arg(G^*(ju) + =) < n/2$, u E (—ro, ro). For simplicity, we set in what follows K = ro. By simple transformation, the results stated for K = ro can be reduced to those meant for a finite K by replacing G(ju) by (G(ju) + ==), and feedback gain by an equivalent gain after an appropriate transformation.

Perhaps the most beautiful result for nonlinear time-invariant feedback gain systems, with a first-andthird quadrant non-monotone nonlinearity¹³ <^(-) in the feedback path, is the celebrated criterion of Popov [2]: !R(1 + jau)G(ju) > 0,0 < u < ro, where a > 0 is a constant. A vast number of stability results have appeared over close to twenty-and-odd years post-Popov in the form of considering monotone and other subclasses of nonlinearities in exchange for weakening the restrictions on the phase angle behaviour of G(ju). With this brief background, it is time to present my main contributions as applied to the *stability/instability analyses of (linear and) nonlinear time-varying feedback gain systems*.

The first major (and early) result [4] for the exponential stability of linear time-varying feedback gain systems was obtained in an extended Lyapunov framework by solving a differential inequality (in the manner of C. Corduneanu) and reads as follows: If the linear time-invariant feedback gain system obeys the standard Nyquist criterion with an *effective* damping factor, and the integral average of the positive lobes of $\{d(t) = \frac{k(t)}{k}, \text{ represented by } 9^+(t)\}$, is bounded for all finite time, but its asymptotic limit (i.e., as the interval of integration tends to ro) is bounded from the above by 2fl, then the system is exponentially stable.

Similar results have been obtained for the exponential stability of finite-dimensional nonlinear timevarying feedback systems (i.e., Case (b) above), with non-monotone and monotone nonlinearities, in the extended Lyapunov framework and once again by solving a differential inequality. However, the most general early result [5] for the L₂-stability of nonlinear time-varying feedback gain systems (i.e., Case (b)) with a monotone nonlinearity was obtained by an amalgamation of Zames's positive theory and Popov's Fourier transform-based framework. The multiplier function, which is a general causal + anticausal function,¹⁴ is due to O'Shea [6], and Zames and Falb [7]. The novelty of the stability criterion is three-fold: (i) The inverse Fourier transform of the multiplier function needs to satisfy an exponentially weighted integral inequality, with (as the exponent for the anticausal function, and £ as the exponent for the causal function. The exponent £ determines the global (as t ^ ro) lower bound on the integral average of the negative lobes of 9(t) (the latter being represented by $9^{-}(t)$), and the exponent £, the global (as t ^ ro) upper bound on the integral average of $9^+(t)$. The finite-time averages of the integrals of $9^-(t)$ and $9^+(t)$ need merely to be bounded. (ii) We can trade £ with £. (iii) If we choose f = f, then, for odd-monotone nonlinearities, there is no restriction on 9(t), which implies that the stability criterion is effectively the circle-criterion for odd-monotone nonlinearities, but with a general multiplier function subjected to a constraint on the weighted integral of the inverse transform of the multiplier function.

No such results are known in a Lyapunov framework (as applied to finite-dimensional systems), which also seems to imply that no counterparts of the Kalman-Popov-Yakubovich lemma [8, 2, 9] for

asymptotically stable; and in the framework of positive-operator theory, L2-stable.

¹³This is the class N. We write for simplicity $y(\pounds E N)$.

¹⁴For brevity, this is called hereafter the OZF multiplier function in honor of its originators.

general multiplier functions have been established. Even if one succeeds in discovering such a counterpart, its utility seems only to be non-stability domains, involving *mere* applications of the putative generalizations of the so-called linear matrix inequalities (LMIs). More explicitly, a counterpart may not lead to any superior or new stability conditions for the problem under consideration.

The most recent result [10], while being a significant improvement over [4] but not entirely resolving the necessity-and-sufficiency conundrum of the Popov theorem, is the first ever result for the Testability of a nonlinear single-input-single-output (SISO) feedback system, described by an integral equation and with the forward block transfer function G(juf) and a first-and-third quadrant (i.e., to repeat, class N, non-monotonic) nonlinearity in the feedback path, but employing the OZF multiplier-function.¹⁵ In this sense, it is an improvement over the Popov theorem¹⁶, and is, interestingly enough, a veritable climax of my earlier results to be outlined below for the sake of completeness.

More explicitly, the improvement consists in the employment of the OZF multiplier function whose timedomain L₁-norm is constrained by certain characteristic parameters (CPs) of the nonlinearity obtained from certain novel algebraic inequalities. If the nonlinearity is monotone or belongs to any prescribed subclass of N, its CPs are reduced, thereby relaxing the time-domain constraint on the multiplier. An important special feature of the new stability results is a partial bridging of the significant gap between the Popov criterion and the stability results that appeared post-Popov in the form of considering monotone and other subclasses of nonlinearities in exchange for weakening the restrictions on the phase angle behaviour of G(ju). Extensions to time-varying nonlinearities more general than those in the literature are also presented. The background and motivation for such a surprising result merit some recounting of research efforts and interactions.

1.2 Background and Motivation for the Latest Result [10]

Early 1970, while in Moscow, I met the doyen of nonlinear stability analysis, Professor Mark Aizerman, at the then-called Institute for Automation and Telemechanics (now renamed as The Institute for Problems of Control), who is better known for his conjecture [11] than for his beautiful monograph [12]. The purpose of my visit to him was to discuss certain issues related to his junior colleague (or assistant or whatever was appropriate in those days), Dr. E. C. Pyatnitskii, whose paper on the *absolute stability* of nonlinear time-varying systems had just then appeared.¹⁷ After a brief exchange of pleasantries, Professor Aizerman declared (in Russian) that research in stability analysis, according to him, was "Ne modna", which means not fashionable.¹⁸ He had by then already *migrated* to path-breaking work on pattern recognition in which he is known for the *potential function* method (the "kernel trick").

Now fast forward to the year 2004 when I found myself in the National University of Singapore (NUS) where, among other things, I started interactions with the active group on control. In course of time, while participating in seminars (on control) at the NUS, given by visiting researchers

¹⁵Note the striking contrast with the Popov multiplier function (1 + jaw) already mentioned above.

¹⁶A possible generalization of the Popov theorem on the lines of the results of [10], i.e., with $\langle \pounds \rangle$ (•) G N, but with no characteristic parameters of the nonlinearities entering the time-domain constraint on the multiplier function, has been conjectured in the same reference. Another conjecture made in [10] is a similar generalization of the results of [5].

¹⁷See item (3) in *Notes* at the end of this article.

¹⁸See item (4) in *Notes* at the end of the article.

(mostly from the US), I presented (during the discussion period) some unresolved stability (and stabilityrelated) problems, including certain results which I had discovered to be faulty.¹⁹ By and by, discussions with the research group on control led gradually to the derivation of new ^-stability conditions for (oddmonotonic) nonlinear periodic-coefficient (i) single-input-single-output (SISO) [13]; and (ii) multi-inputand-multi-output (MIMO) [14] systems. The former paper is a consolidation and detailed computational verification of [15]; and the latter, which entails the solution of certain novel matrix-algebraic time-varying linear and nonlinear inequalities, constitutes a significant improvement over the results of the literature, by assuming no symmetrical time-varying feedback gain matrices, along with class N vector nonlinearities.

It has turned out that, for discrete-time linear and nonlinear time-varying feedback gain MIMO systems, the testability conditions are not mere *discretized* versions of those for continuous-time MIMO systems presented in [9]: for instance, the generalized circle-criterion of the former has no counterpart in the latter. A striking byproduct is that when the results of discrete-time MIMO systems [10] are specialized to be applicable to discrete-time SISO systems, the ^-stability conditions significantly generalize those of [11], by, for instance, being applicable to non-monotonic nonlinearities with the OZF multiplier function, thereby surpassing all the earlier results of Tsypkin, Jury and Lee, O'Shea and others. For continuous-time systems, the paper [10], mentioned in the last subsection, is a culmination of recent efforts.

Aberrations in the Derivation of Stability/Instability Results

Lest one should lose sight of the forest for the trees, one needs to note that all the stability results have been obtained under the assumption (called for brevity **Assumption A1**) that the **basic linear timeinvariant system with feedback gain** K e [0, K) is asymptotically stable, as determined by applying the Nyquist criterion or the Routh-Hurwitz conditions. Under Assumption A1, I have established even L_p stability conditions for linear and nonlinear time-varying feedback gain systems using the Riesz-Thorin interpolation theorem. However, all such the stability conditions are only sufficient. It is not known how to establish their necessity.

Analogously, most of the instability results of the literature invoke (in some form or the other) the method of contradiction to establish instability, but the end results turn out to be mostly either impotent or somewhat faulty [16]. The strange thing is that the derivation of instability conditions for linear and nonlinear time-varying feedback gain systems is as though inextricably bound up with the assumption (called for brevity **Assumption A2**) that the basic linear time-invariant system with feedback gain K e [0, K) is unstable, as determined by applying the Nyquist criterion or the Routh-Hurwitz conditions. Set against such a background, I have developed a constructive method to establish instability conditions for linear and nonlinear time-varying feedback gain systems using a not-so-well known converse of the Cauchy-Schwartz inequality [17]. Further, invoking the converse Holder inequality, I have derived L_p -instability conditions for the same systems [18].

The unresolved problems beckoning to researchers are: (i) With Assumption A2 holding, replace the gain K by the time-varying function $k(t) \in [0,K)$, t > 0. Do there exist *global constraints* on k(t) and/or its derivative to stabilize the system? (ii) Again, under the same assumption, suppose the gain K is now replaced by the time-varying nonlinear function <^(-)k(t), with <^(-) e N; k(t) e

¹⁹See item (5) in *Notes* at the end of the article.

[0, K), t > 0; and $-1 \mid_x = 0 < 1$. What are the global constraints on k(t) and/or its derivative to stabilize the system?

The corresponding counterparts for Assumption A1 are obtained from the last paragraph by substituting the word "destabilize" for "stabilize".

2 Second Phase of Research at the Institute

The drastic change in my research interests was also inaugurated by Professor B. L. Deekshatulu who did not like my "getting lost in the unreal world of symbols". He had returned from IBM Labs in the US in early 1972, with stimulating ideas related to image scanning and processing, as also applications of image processing to remote sensing of agricultural areas by aerial color infrared (CIR) photography. Possibly inspired by Professor P. R. Pisharoty's work (in Kerala) on identifying coconut tree diseases from aerial photographs, and his own first-hand exposure to remote sensing experiments in Michigan which he visited during his US sojourn, Professor BLD managed to get financial support (around 1975/76) from (if I remember correctly) Space Applications Center (SAC), Ahmedabad — ISRO had not yet come into existence at that time in the present form — for aerial CIR imaging of sugarcane crops in Mandya (near Mysore) by mounting Hasselblad cameras (borrowed from SAC) on the Institute's Pushpak aircraft. There was, of course, a co-investigator from SAC, too. Since the color scanner needed for conversion of CIR images to digital format was not ready, we had to resort to manual interpretation of a large number of images.

In the middle (or so) of 1976, Professor BLD joined the National Remote Sensing Agency in Hyderabad. I was left alone to set up the Remote Sensing and Image Processing Laboratory, in which all the manual interpretation of the so-called Mandya Project CIR images (of sugarcane and paddy crops) was accomplished, along with the first attempts to develop an blue-green-red²⁰ LED- based scanner. In the project funded by ISRO, we developed all the basic software (in Fortran) on HP 1000 (which had only 10 KB, yes 10 KB solid-state memory, and no tape drive²¹), for image processing and pattern recognition (for resources estimates on a small scale). For any meaningful computational work, we had to reserve terminal time at the Computer Center which had a DEC machine, but no image display of raw and/or processed image data was possible. This necessitated the development of our own color image display using the Conrac monitor which I had managed to procure after more than a year of struggle with the formalities of 'clearances'.

All such formalities had been successfully completed for the purchase of a Printronix dot-matrix printer (which was originally meant for a planned PDP 11/34, instead of the HP 1000²²). It was felt unwise to cancel its shipment from the US, fearing loss of time (and hence a possible refund of the sanctioned money to ISRO). Once the printer arrived, we had to develop appropriate interface (driver- software) between HP 1000 and Printronix.²³ In all these ground-breaking activities, the crucial contributors were Mr. Animesh Mukherjee (who had registered for an M. Sc. (Res) degree) and Mr. William Raymond.²⁴ I cannot adequately praise their whole-hearted devotion and achievements. The

²⁰See item (6) in *Notes* at the end of the article.

²¹ This was because the Electronic Commission limit for Institute's purchases, Rs 5 lakhs, without the so-called 'global tender' to be floated and decided on by the Government itself.

²²This computer had no built-in interface for Printronix.

²³The HP representative in Bangalore showed interest in 'buying' our Printronix driver software for HP.

²⁴See item (6) in *Notes* at the end of the article.

Computer Vision and Artificial Laboratory (along with its subsidiary, Remote Sensing Laboratory) that came up in course of time owed — yes, it is past tense — its existence primarily to them. And this place of my work in the Institute was literally my *sanctuary* until 2004.

2.1 The Subjective World of Image Processing

Any serious and thoughtful researcher will find that most of the literature in image processing contains purely *ad hoc* methods, with the number of papers on, for instance, edge detection running to thousands. In the laboratory we did implement quite a few of them, and always disagreed on how useful they are in the context of classification of, for instance, remotely sensed data. In fact, it would be quite relevant to know what an edge is. Is a 'mathematical edge' (as found in various books) applicable to what we, as humans, perceive something in an image as an edge? Is the much-harped-upon edge detection an end itself, implying thereby that we do not need to do anything else to an image in order to extract information from an image? In my view, no! I am sure this could stir up an acrimonious debate among researchers still trying to write up yet *another* paper on edge detection, as Professor Azriel Rosenfeld would declare.

Professor David Marr, the author of the remarkable and influential book, *Vision*, was perhaps the first to attempt modeling the human visual mechanism for detecting boundaries among various objects in the image of a scene as cast upon the retina. Inspired apparently by the seminal work done by the Nobel Laureates, Professor David Hubel and Professor T. N. Wiesel, on the visual mechanisms of a mammalian visual system, and guided by the empirical findings on the bandpass characteristics of the (mammalian) visual system as obtained from *flashing* suitably calibrated bar-like patterns to it, Professor Marr proposed the existence of detectors of extremal points of gradients of intensity changes (in the visual scene) in the outputs of three/four bandpass channels, which were, in turn, obtained by using Gaussian filters with appropriately chosen variance parameters (to achieve a spread of an octave for each channel and partially overlapping with each other). He called them zero-crossing detectors. In effect, the human visual cortex stores the visual scene in the form of boundaries of objects in the scene, and is able to recall the scene, as it were, in our imagination, just on the basis of those boundaries. After all, the number of neurons in the human brain is limited, and the brain cannot possibly store all the information in a scene.

Now comes the mathematical problem (**Problem 1**): Is it possible to construct an image of a scene using merely these multi-channel of octave-width (or otherwise) zero-crossing points (which when theoretically assembled are supposed to give us contours of objects in the scene)?

Another, equally important problem (**Problem 2**) is this: Assuming that the boundaries of objects contain all the required information available from a scene, how can we automatically recognize the objects in the scene?

Noting that humans have been endowed with two eyes that facilitate depth perception, a related problem (**Problem 3**) is to automatically estimate a scene's depth information from the stereoscopic pair of images of the scene by extracting the boundaries²⁵ of objects from each image of the pair.

The above three mathematical problems acted as some sort of guideposts for work in the laboratory.

²⁵The assumption is that boundaries of objects are more reliable sources of information than the (raw) gray/color intensities of the images of the scene. This is because images are naturally affected by noise, but boundary detection operators are supposed to be less susceptible to noise.

The extensive work that is presently being done all over the world in image processing, vision and pattern recognition is an indication that no simple account can be presented here so as to convince a reader of the complexity of the problems under consideration and subjectivity surrounding each result. Note that there is absolutely no chance of mathematical purity and clarity of the type found, for instance, in the first part of this article. Moreover, any major description of the work I did (or was involved in) first in the Institute until 2004-05, and second, outside Bangalore subsequently, would require a veritable treatise which is not in order here. Therefore, only a bird's eye-view is provided below to give a glimpse of some contributions.

• Problem 1.By decomposing an image to scanned lines, I developed a new framework consisting of generalized Hermite polynomials, and dispensed with the standard, but unrealistic, assumption of bandlimitedness of (spatial) signals. The motivation came from Slepian's paper [19] in which he has introduced the concept of "effective time-spread" and "effective frequency- spread" for functions of time. This translates to corresponding spreads in the space-frequency domain for images.

By exploiting the special properties of Hermite polynomials, it is shown [20] that a unique reconstruction (except for scale factor) of a scanline is possible from its (real) zero-crossings,²⁶ given its space-bandwidth product or space-bandwidth ratio. Extension to images (considered as stacks of scanlines) involves space-bandwidth products or ratios (including cross-space- bandwith products/ratios) in two dimensions [21]. The same framework can be used to reconstruct signals from other types of partial information of the type (i) Fourier phase only; and (ii) Fourier magnitude only. For an entirely different approach, involving the concept of a regularized solution for the reconstruction of images from points given arbitrarily near boundaries and gradient information at these points, see [22].

Pro tem moving forward to the post-2004 period at the NUS (Singapore), during discussions (with the signal processing group) on compressive sampling — a very fashionable topic these days — I came across a more striking problem of reconstruction of assumed sparse digital (sampled in time, for instance) signals²⁷, from partial information sampled from, say, its Fourier transform. During some preliminary studies on this challenging topic, I happened to across a paper in which the so-called Gini-index (which originated in economics) was being used to characterize sparsity. Invoking such a definition and using the (discrete) \pounds_q -norm (with 0 < q < 1) along with simultaneous perturbation stochastic optimization (SPSO), we came out with a more efficient and more accurate reconstruction algorithm [23]. Note that this has no counterpart in the modelling of human visual system because it seems most unlikely that the human brain stores compressive samples of images of scenes for reconstruction during recall.

• Problem 2. We first explored many algorithms of the literature on two-dimensional (unoccluding) pattern detection and recognition. The process involved extraction of boundaries

²⁶What are complex zero-crossings as applied to an image? Assuming that the zero-crossings in an image are obtained as maximal/minimal gradient points of a Gaussian-filtered — to simulate multiple (octave-width?) bandpass channels of the human visual system — image, how does one label them as real and complex zero-crossing points, if any? These are questions which seem to be still in need of clarification.

²⁷Sparsity is normally expressed in terms of the number of elements in the signal having either zero magnitude or negligibly small amplitudes.

which were most of the time broken, thanks to the inevitable use of thresholds at various stages of processing. We did invoke ingenious tricks suggested by a few authors (incl. Azriel Rosenfed) to 'close' the broken boundaries, and then employed strategies (like relaxation labeling) to segment the image. None of these were satisfactory in our preliminary goal of detecting even a card-board cube in a synthetic lab scene. To cut the story short, disenchantment led to my foray into an application of artificial neural networks, and, in particular, the multilayer perceptron (MLP).

The first major study was an attempt to implement Fukushima's Neocognitron, a neural network to recognize characters. After a significant number of experiments with it to identify English alphabet and numerals, we found it to be unsatisfactory. This is the origin of our paper [24] which presents a modification of the Neocognitron.

The standard MLP treats a two-dimensional pattern as a stacked 1-D vector, and hence two similar patterns shifted by even one-pixel are regarded as two different patterns. For the same reason, invariance to scale and distortion cannot be achieved satisfactorily. Inspired by the retinal structure of the human visual system, we were motivated to develop a radial encoding of two-dimensional patterns which was successfully used to recognize patterns of non-overlapping characters. The first step was to generate synthetically a large number of patterns of each of the characters subjected to distortions (within limits) in scale and rotation, and also affected by controlled amounts of noise. These patterns were then fed to Kohonen's self-organizing neural network (SONN) by way of checking the possibility of finding clusters (in feature space) for the characters. Excited by the output of the SONN, we developed some efficient algorithms not only to recognize characters and numerals, but, with some novel modifications depending on the applications, to boundary/contour extraction, face detection and recognition, multispectral data classification, among others [25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35].

Lest one should get the impression that the problem of face (detection and) recognition from images has been resolved satisfactorily in the ever-growing literature, it may be startling, but advisable, to note here that none of the algorithms reach the remarkable efficiency and accuracy of the (trained, wherever required) human visual system. The same is true of the classification of human facial emotions from images. Even with three-dimensional facial data, success with recognition of faces and their emotions has not been satisfactory.

Application of neural networks to three-dimensional face recognition is quite a complex issue, apart from the fact that a huge database needs to be created and processed. While processing a publicly available three-dimensional database (of limited size) of human faces, it was found that the data points were irregular samples, and were not amenable to direct analysis for face (and facial expression, as one of eight basic emotions) recognition. After applying a data resampling strategy, we have developed a modified form of principal component analysis to recognize (three-dimensional) faces and their expressions [36, 37] with an accuracy better than what is reported in the literature. The fact is that the same approach (after all the necessary modifications or changes) gives highly unsatisfactory results on other databases. A universal solution to the problem of face (and facial expression) recognition, independent of the origin of the datasets, is most unlikely anytime soon.

• Problem 3. It is known that stereoscopic image analysis deals with the reconstruction of

a three-dimensional physical scene (containing objects of interest) from at least a pair or, as is currently the practice, multiple images of the scene captured from different locations. Most of the literature contains (area-based, feature-based, diffusion-based, Bayesian and the like) algorithms for matching *corresponding* points in the images under the assumption of epipolar geometry,²⁸ the points being the projections of the same scene point on the images. Assuming that the geometry of image acquisition is given, the map of matched points can be used to compute the depth information in the scene. When applied to calibrated scenes, these algorithms have been found to be generally unsatisfactory. Even artificial neural networks have been tried out.

Motivated by the self-organizing (i.e., so-called unsupervised) model proposed by Kohonen to imitate perceptual mechanisms of the human brain, we employed a modified self-organizing network to match the corresponding points in a stereo-pair of images. Quite distinct from the existing algorithms which, typically, involve area- and/or feature-matching, the network is first initialized to the right image, and then deformed until it is transformed into the left image, or vice versa, this deformation itself being the measure of disparity. This novel approach, which dispenses with the standard assumption of epipolar geometry, has been successfully tested on synthetic and natural (including random-dot stereograms and wire frames) and distorted stereo-pairs [38]. Such an application of the principle of self-organization to extract depth information in a scene from stereo-pairs of images seems to be the first of its kind.

• Other Research Contributions: Apart from the above, additional work was done in the laboratory on certain aspects of image representation (quadtrees), optical flow, image fusion and modeling the human visual system. These topics do not, in general, relate to what was presented above as three basic problems. Since it would take a significant amount of space to describe these contributions, I thought it advisable to merely list them as [39, 40, 41, 42, 43, 44], while hoping that their titles give an idea of their contents.

I wish to end my article with a reference to Turing's much-debated paper (1950) [45] with the title, *Can a machine think*?

As is well known, lots of issues engendered by an interpretation of Turing's question have not yet been settled satisfactorily. In 1950, Turning declared: "I believe that in about fifty years' time it will be possible to programme computers, with a storage capacity of about 10⁹, to make them play the imitation game so well that an average interrogator will not have more than 70 percent chance of making the right identification after five minutes of questioning. I believe that at the end of the century the use of words and general educated opinion will have altered so much that one will be able to speak of machines thinking without expecting to be contradicted." Many mathematicians and philosophers are of the opinion that his claim has been shown to be mistaken. See, for instance, [46, 47].

With the above formidable history related to thinking machines, can we venture to ask for a machine that can "see"? Or, is it the case that *thinking* and *seeing* are analogous processes. If a machine can think, can it also see, provided it is subjected to appropriate inputs? Answers to

²⁸ A assumption of epipolar geometry implies that the corresponding points lie on the same scanline in the stereo-pair under consideration.

these questions could hopefully trigger a stimulating debate among those interested in classical-vs- modern-AI (artificial intelligence) controversy. As a first step to grapple with the imbroglio, what tests, involving the use of cameras and computers, are to be designed to set goals for the realization of a possible visionmachine? Are there arguments analogous to, for instance, Searle's *Chinese Room* [46] as fundamental objections to the very concept of a vision-machine?

3 Epilogue

I wish to place on record my deep gratitude to my teacher, mentor and father-figure, **Professor BLD**, for his ever-benevolent association with me all these years, and to **Professor M. A. L. Thathachar**, my former teacher and later colleague, for his crucial role and providential help rendered in the early stages of my work in stability analysis.

While I consider the life I spent at the Institute to be the most glorious and occasionally ecstatic and I would be willing to live it once again if there were an opportunity, I cannot but end these reminiscences without quoting William Blake's poem, *The Defiled Sanctuary:*

I saw a Chapel all of gold That none did dare to enter in, And many weeping stood without, Weeping mourning, worshipping. I saw a Serpent rise between The white pillars of the door, And he forc'd and forc'd and forc'd; Down the golden hinges tore, And along the pavement sweet, Set with pearls and rubies bright, All his shining length he drew, Till upon the altar white Vomiting his poison out On the Bread and on the Wine. So I turned into a sty, And laid me down among the swine.

Notes

1. There entered, in the early 1970s, a new, foreign import to the Indian Institute of Science (IISc), at the level of the highest academic position. This event was a very precious fruit of the efforts of a special committee set up in the late 1960's which literally scoured the world, interviewing candidates with the goal of bringing in "specialists from abroad", to ostensibly

"avoid inbreeding" and, at the same time, earnestly hoping that frontier research work would be launched by them at the Institute. This person of significance (with apologies to Nikoloi Gogol, the Russian novelist), soon after coming over to the Institute from a place not exactly on the map of distinguished places of higher learning, began his activities by forcing himself on junior members of faculty who were teaching graduate courses, and sat in some of the classes — much to their chagrin and disgust — as though to grade them. He used to pass unsolicited judgement on their teaching standards. In one such case, in fact, when he felt his blatant claim as the sole authority on a certain course-subject was in question, he *wrested* the "custody", as it were, of that course, wagged his first finger (as teachers used to do in middle school in those days) at the hapless instructor, and verbally threatened him with dire consequences on his promotion.

There were other unpleasant events associated with the same person. Once, in his seminar, which was widely announced, he wrote on the blackboard some mathematical expressions. A mathematician of high caliber in the audience was bold enough in bringing to the speaker's attention inaccuracies in those expressions. The speaker's fury stunned the audience, especially when he shot back: "Have you worked on the problem?"

- 2. I have recently discovered that a much-quoted and beautiful stability result of Lyapunov [48] when applied to Eqn. (1) does not *seem* to exhibit the set of stability regions in the *a-vs-0* graphs of [1]. More simply, there are (a, P) values inside the McLachlan stability boundaries which do not obey the Lyapunov condition for stability. I wish to add, however, that I am yet to access M. G. Krein's (Russia) papers (on the type of differential equation considered by Lyapunov [48]), and collected in AMS Translations, which are supposed to be an improvement over Lyapunov's result.
- 3. I was told about this paper by another well known researcher (from the same Institute), Professor Ya. Z. Tsypkin, the author of a book on adaptive control, as also of a paper in Doklady Academii Nauk SSR (around 1965 or so) on the extension of the Popov theorem to discrete-time nonlinear time-invariant feedback gain systems. (This paper will figure in later in the article.) I used to have regular meetings with him in Moscow either before or after his lectures on learning systems. Once he gushed with pride to tell me that Dr. Pyatnitskii had applied Pontryagin's maximum principle to solve a stability problem. When I responded by pointing out that Professor Roger W. Brockett had already done (or rather attempted) it see item
 - (5) below in 1964, he was unimpressed.
- 4. For the sake of completeness, I wish to add, however, that at the end of those discussions on the Pyatnitskii resolves in disagreement. By the way, *control* as a discipline in India began the steep descent around this time. Except for some indirect references to control in power electronics and the like, there seems to be no visible research interest now in India in control theory and practice per se. Contrast it with what the Russians have done to Aizerman's institute they have christened it the Institute of Problems in Control.
- 5. The fact that it [49, 50] is a faulty result I discovered [51] in early 1975 when I was at the University of Karlsruhe, Karlsruhe (Germany), interacting with my German colleague, Dr. Gerhard Siffling, on the stability analysis of a discrete-time FSK oscillator. It is appropriate

to mention here that Brockett did not *actually* employ the Maximum Principle for the system governed by the linear second order differential equation, df + 2d + k(t)x = 0, where the time-varying gain k(t) > 0, t E[0, ro). His technique, in fact, amounted to choosing k(t) for t > 0, such that the phaseplane trajectory starting from, say x(0) = 0, $df|_{t=0} = 2$, results in the *limiting-case* of a closed loop trajectory having the largest intercept on the x— axis. In this context, it seems pertinent to pose the following problem related to a possible application of the Maximum Principle to the Mathieu equation whose stability/instability boundaries in terms of a vs β plots are known, as mentioned earlier: What is a typical performance criterion to be maximized in terms of parameters a and β ? In this context, see [16] for other instances of faulty results, including the one in which what is to be

In this context, see [16] for other instances of faulty results, including the one in which what is to be proved is assumed. And, interestingly enough, this last faulty result is the one which has been *extended*, by a researcher, to Marcinkiewich space without even checking its correctness.

6. While in Germany in 1975, I had managed to contact the division of Siemens where blue and green LEDs were being manufactured for the first time. They were kind and generous enough to send me gratis a couple of their samples. Along with the Hewlett-Packard HEDS unit, the standard red LED and the blue and green Siemens LEDs were ingeniously assembled by my newly-acquired assistant, Mr. William Raymond (who had just a diploma in electronics) and mounted on the movable carriage of an old lathe in the department. The spindle of the lathe provided the rotary motion for a cylindrical drum on which a CIR image print could be stuck. And so was created the first solid-state scanner in India. We did succeed in scanning a couple of black-and-white and color images just to demonstrate the possibility of conversion of a hard copy color image to digital format.

There is a lot more of bumpy history associated with the handling of the LED scanner output which cannot be presented here. Thanks to the facetious remarks by some members of the project monitoring committee, no refinements of LED scanner for research purposes could be attempted. A typical remark was that it was simpler to buy an optronics scanner from the US or a laser scanner from France! That was a time when anything to be bought from abroad had to be cleared by the Electronics Commission. By the time any equipment arrived in India (which would have taken not less than two years), the model would be outdated, as it did in fact happen with respect to the Hewlett Packard computer HP1000 which required superhuman efforts just to get formally cleared by the various authorities of the Government. By the time the unit arrived at the Institute, the model (with an older technology disk drive that was very fragile) was already outdated. By the way, it was the first computer in the department and this was bought from ISRO project funds for image processing. Getting back to the story of the laser scanner imported by a Government agency, when that scanner stopped functioning, it could not be repaired for more than one year. We subsequently learned that the unit soon fell into disuse.

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